Improving Human Perception in Bilateral Teleoperation and Haptic Interaction
Teleoperation Systems and Haptic Interfaces
Classical Control Schemes

FORCE REFLECTION

FORCE REFLECTION WITH PASSIVITY

POSITION REFLECTION
Control algorithms: Force Reflection

Model of master and slave:

\[
M_m \ddot{x}_m(t) + B_m \dot{x}_m(t) = F_h(t) - F_m(t)
\]

\[
M_s \ddot{x}_s(t) + B_s \dot{x}_s(t) = F_s(t) - F_e(t)
\]

Control equations:

\[
F_m(t) = F_s(t - T)
\]

\[
F_s(t) = -K_s [x_s(t) - x_m(t - T)]
\]

Sufficient condition for stability and asymptotic tracking (Spong-Ortega 2003):

\[(2^{0.5} - 1) \frac{B}{K} > T\]
Control algorithms: Position Error

Model of master and slave:

\[ M_m \ddot{x}_m(t) + B_m \dot{x}_m(t) = F_h(t) - F_m(t) \]
\[ M_s \ddot{x}_s(t) + B_s \dot{x}_s(t) = F_s(t) - F_e(t) \]

Control equations:

\[ F_m(t) = -K_m [x_m(t) - x_s(t - T)] \]
\[ F_s(t) = -K_s [x_s(t) - x_m(t - T)] \]

Sufficient condition for stability and asymptotic tracking (Spong-Ortega 2003):

\[ B_m B_s > T^2 K_m^2 \]
Teleoperation in terms of Dirac Structures

- $\mathcal{D}_m$ and $\mathcal{D}_s$ represent the master and the slave device
- $\mathcal{D}_{cm}$ and $\mathcal{D}_{cs}$ are the master and the slave IPCs
- $\mathcal{Z}$ represents the data-transmission blocks
Energetic Analysis

DATA COMM.

\[ M_m : l \]
\[ F_h \]
\[ v_h \]
\[ B_m : R \]
\[ M_{Se} \]
\[ F_{mc} \]
\[ x_m \]

\[ T \]

\[ F_{sc} \]
\[ M_{Se} \]
\[ F_e \]
\[ v_e \]

\[ B_s :: R \]
\[ M_{Se} \]
\[ x_s \]

\[ M_s :: l \]

\[ \text{USER} \]

Nicola Diolaiti

CASY – 01/17/2005

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Intrinsically Passive Controllers

- Passivity-based techniques allow to preserve the stability of the overall system.
- If a controller is equivalent to a physical (passive) system, no extra-energy is injected into the plant.
Scattering and Impedance Matching

To preserve the stability of the overall system in presence of time delays in data transmission between master and slave, flows and efforts are encoded in a power-consistent way:

\[
\begin{align*}
\frac{1}{\sqrt{2b}} & \quad (e, f) \\
\frac{1}{b} & \quad (e, f)
\end{align*}
\]

Once that the line impedance \( b \) is chosen, a system with the same impedance has to be connected at the power port \((e, f)\) to avoid waves reflections **(impedance matching)**
Scattering and IPC

- Passivity is an intrinsic feature of the control scheme, regardless of $T$

User perception $(e_h, f_h)$ of the remote environment $(e_E, f_E)$ is affected by spurious dynamics due to:

- IPC controllers
- scattering coding
Scattering and IPC: experimental results

Positions

Interaction forces

IPC Forces

Xm / Xs

Fmr / Fsr

Fh / Fe

Fm / Fs
Example

- Master – Slave system:
  - Sensable Phantom
  - Industrial 6 dof arm (COMAU Smart 3S)

- Implementation of IPC control scheme with geometric scattering and scaling of variables on the Comau/Phantom system (on going)
  - Driver for ATI force/torque sensor for RTAI-Linux
  - Different communication protocols:
    - CORBA
    - RtNet (RTAI-Linux)
One-Dimensional Teleoperation Scheme

To obtain some initial results, the following simplified one dimensional telemanipulation scheme is adopted:

When the slave is in contact with an object, our purpose is to reflect its stiffness by passively adapting the spring $k_m$.

The adaptation of the damping coefficient $b$ is not possible because it has to satisfy the impedance matching condition.
Passive Parametric Variation

- **Basic idea**
  for a linear spring

\[ \mathcal{H}(k, x) = k \frac{x^2}{2} \]

- **\( \sigma(t) \)** is a parameter that has to be designed to achieve a **desired stiffness** \( K \) at the external port \((e, f)\)

\[
\begin{pmatrix}
1 & 0 & 1 \\
0 & 1 & \sigma \\
0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
\dot{x} \\
\dot{k} \\
f
\end{pmatrix}
+
\begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
1 & \sigma & -1
\end{pmatrix}
\begin{pmatrix}
\frac{\partial \mathcal{H}}{\partial x} \\
\frac{\partial \mathcal{H}}{\partial k} \\
e
\end{pmatrix}
= 0
\]

\[
\sigma(t) = 2 \frac{K - k(t)}{x(t)}
\]
Simulations

- Simple mass/spring/damper system
The modulation of the master and slave IPC controllers has to preserve the passivity.

- No extra-energy has to be injected through modulation ports
- IPC controllers have to be extended including power ports related to variations of parameters
- Suitable interconnections have to be designed
Formalization of the Transparency Notion

We have (still) to find conditions to get the “best matching” between the power port \((e_h, f_h)\) and \((e_E, f_E)\)

✓ Behavioral approach

These conditions will actually lead to the design of the parameters adaptation laws
Application to the Teleoperation Scheme

- The force applied by the user smoothly grows to a fixed value
- The initial distance between the slave and the obstacle E is 1m
- When the slave touches E, the stiffness $k_E$ is estimated

Initially:

$$k_m = k_s = k_0$$

The adaptation law:

$$k_{ref}(t) = k_0 + k_E(t)$$

allows the user to perceive the contact with E, by means of an increased stiffness
Contact Modeling and Estimation

- Overview on Contact Models
  - Coefficient of Restitution
  - Kelvin-Voigt Linear Model
  - Hunt-Crossley Non-Linear Model

- Estimation Algorithm for the Hunt-Crossley model

- Experimental Results
Coefficient of Restitution

According to the Newton model:

\[ v_o = -e v_i \]

Experimentally, it turns out that \( e \) depends on \( v_i \):

\[ e = 1 - \alpha v_i \]

And the dissipated energy is approximated by:

\[ \Delta H \approx \alpha v_i \frac{p_i^2}{m} \]
Kelvin-Voigt Linear Contact Model

- A continuous model relating the contact force to the contact dynamics is desirable.
- The mechanical parallel of a linear spring and a linear damper is the simplest way to represent energy storage and dissipation:

\[ F(t) = \begin{cases} 
Kx(t) + B\dot{x}(t) & x \geq 0 \\
0 & x < 0 
\end{cases} \]

- However, some physical inconsistencies arise: the coefficient of restitution associated to this model does not depend on \( v_i \).
Drawbacks of the Linear Model

- Analysis of the hysteresis diagram and power exchange diagram:

These problems are particularly relevant for soft materials ($B$ large)
Hunt-Crossley Non-Linear Contact Model

- The inconsistencies of the linear model can be removed if the damping coefficient is made dependent on the approach $x$
  - the viscous force is null when $x=0$

- Hunt-Crossley non-linear contact model:

\[
F(t) = \begin{cases} 
K = kx^{n-1} & \text{if } x \geq 0 \\
B = \lambda x^n & \text{if } x < 0 \\
0 & \text{if } x = 0
\end{cases}
\]

- The parameter $n$ depends on the geometry of the contact surfaces; usually $n \in [1,2]$

- The elastic term is consistent with the Hertzian theory of contacting spheres in static conditions ($n=3/2$).
Advantages of the Non-Linear Model

- Analysis of the hysteresis diagram and power exchange diagram:

- Suitable to characterize compliant materials while retaining a certain computational simplicity
Estimation Algorithm

- On-Line estimation of the parameters $k$, $\lambda$, $n$ of the Hunt-Crossley model when the probe is in contact with the unknown material:

$$F(t) = kx^n(t) + \lambda x^n(t)$$

- By taking advantage of the particular structure of the model (linear w.r.t. $k$, $\lambda$, and, with manipulations, linear w.r.t. $n$), the proposed estimator has the following scheme:

$h$ is the discrete time-variable

\[ \Gamma_1(x, \dot{x}, F, \hat{n}) \]

\[ \Gamma_2(x, \dot{x}, F, \hat{k}, \hat{\lambda}) \]

\[ \hat{k}, \hat{\lambda} \]

\[ \hat{n} \]
Estimation of $k, \lambda$

- After $N$ samples, the Recursive Least Square estimator minimizes the cost function:

$$V_N(k, \lambda) := \frac{1}{N} \sum_{h=1}^{N} \varepsilon_1^2(h) \quad \varepsilon_1(h) := \phi_1(h) - [k + \lambda \dot{x}(h)]x^n(h)$$

- Hypothesis: $\langle \mathcal{E}_1 \rangle$ is a zero mean stochastic process

$$\Gamma_1 \begin{cases} 
\hat{\theta}_1(h + 1) = \hat{\theta}_1(h) + Q_1(h + 1) \left[ \phi_1(h + 1) - \mu_1^T \hat{\theta}_1(h) \right] \\
Q_1(h + 1) = P_1(h)\mu_1 \left[ \beta + \mu_1^T P_1(h)\mu_1 \right]^{-1} \\
P_1(h + 1) = \frac{1}{\beta} \left[ I - Q_1(h + 1)\mu_1^T \right] P_1(h) 
\end{cases}$$

$$\mu_1 = [x^n(h + 1), x^n(h + 1)\dot{x}(h + 1)]^T$$

$$\hat{\theta}_1(h) = [\hat{k}(h), \hat{\lambda}(h)]^T$$

$\beta$: forgetting factor
Estimation of $n$

- By manipulating the H-C model: 
  \[ \varepsilon_1(h) := \varphi_1(h) - [k + \lambda \dot{x}(h)] x^n(h) \]
  \[ \log \varepsilon_2(h) = \log \frac{\varphi_1(h)}{k + \lambda \dot{x}(h)} - n \log x(t) \]

- Where: 
  \[ \varepsilon_2(h) := 1 + \frac{\varepsilon_1(h)}{[k + \lambda \dot{x}(h)] x^n(h)} \]

- **Assumption**: $\varepsilon_1$ small and independent wrt $F(h)$
  \[ \log \varepsilon_2 = \log \left( 1 + \frac{\varepsilon_1}{[k + \lambda \dot{x}] x^n} \right) \approx \frac{\varepsilon_1}{[k + \lambda \dot{x}] x^n} \]

Then $\langle \log(\varepsilon_2) \rangle$ is a zero mean stochastic process and $n$ can be estimated with a RLS algorithm
Considerations on the feedback interconnection

- Each estimator behaves as a source of additional noise.
- New expressions for the estimation errors have to be considered:
  \[ \varepsilon'_1(h) = \varphi_1(h) - (k + \lambda \dot{x}(h)) x^\hat{n}(h) \]
  \[ \log \varepsilon'_2(h) = \log \frac{\varphi_1(h)}{\hat{k} + \hat{\lambda} \dot{x}(h)} - n \log x(h) \]

- By computing the series expansion:
  \[ \log \varepsilon'_2 \simeq \frac{\varepsilon_1 - x^n(\delta k + \delta \lambda \dot{x})}{x^n(k + \lambda \dot{x})} \]
  \[ \delta \lambda = \hat{\lambda} - \lambda \]
  \[ \delta k = \hat{k} - k \]

- Hence \( \langle \log(\varepsilon'_2) \rangle \) is a zero mean process if:
  \[ \| \delta k + \delta \lambda \dot{x} \| \ll \left\| \frac{\varepsilon_1}{x^n} \right\| \]

At the beginning of the estimation process \( \delta k \) and \( \delta \lambda \) can be substantial but \( x \) is small!
Contact Impedance Estimation: Laboratory Setup

Adaptive windowing velocity estimation from high resolution linear encoder

Linear actuator

Force sensor
Considerations on the motion profile

- The convergence of the estimation algorithm has to be independent on the trajectory of the probe device.
- A sinusoidal trajectory is used ⇒ minimum level of “excitation” to the estimator, but still sufficient because of the non-linear nature of the Hunt-Crossley model.
- Evaluation on the parameters space:

ideal case

real case
Experimental results: HC model, stiff material

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**Graphs:**

- Top left: Force vs. displacement plot.
- Top right: Stiffness parameter ($k$) plot.
- Bottom left: Power output vs. time plot.
- Bottom right: Efficiency parameter ($n$) plot.
Stiff Material - Random Motion
Experimental results: KV model, soft gel
Experimental results: HC model, soft gel
From Soft Gel to Plastic Layer
Soft Gel - Random Motion

Nicola Diolaiti
CASY - 01/17/2005
Technological issues in Force Feedback devices

- Once the mechanical impedance is estimated, it is necessary to adapt the controller of the master device.

- The digital implementation of IPC controllers has to be analyzed in order to preserve passivity.

- It has been shown (Colgate, 1994) that the maximum achievable stiffness preserving passivity is:

  \[ K_{\text{max}} < \frac{2b}{T} \]

- But in real devices intrinsic viscous friction is very small, e.g. PHANToM 1.0:

  \[ b = 0.005 \text{ Ns/m} \]
  \[ T = 0.001 \text{s} \]
  \[ K_{\text{max}} \approx 1000 \text{ N/m} \]
Model of the Haptic Interaction

- Simplified model of haptic interaction:

- Block scheme representation:
Non-idealities in haptic rendering

- The human perception of the Virtual Environment is affected by:
  - Device inertia ($m$)
  - Device intrinsic friction ($b,c$)
  - Sampling frequency ($1/T$)
  - Position Resolution ($\Delta$)
  - Computational Delays ($t_D$)
  - Limited amplifier’s bandwidth ($A(s)$)

- the commanded force is not actuated immediately

CONTROL ALGORITHMS:
- minimize spurious device dynamics and unstable behaviors

NOT AFFECT STABILITY

AFFECT STABILITY
Model of a 1 DOF device

\[ m\ddot{x}(t) + b\dot{x}(t) + c\text{sgn}\dot{x}(t) = F_H(t) + F_A(t) \]

- If we neglect the computational and amplifier delays:

\[ F_A(t) = -K\Delta \left( \left\lfloor \frac{x(hT)}{\Delta} \right\rfloor + \frac{1}{2} \right) \quad \forall t \in [hT; (h + 1)T[, \ h \in \mathbb{N} \]

- We want to relate the stiffness \( K \) of the virtual environment to the non-idealities in the haptic interface.
Dimensionless Parametrization

<table>
<thead>
<tr>
<th>Signal</th>
<th>Dimensionless Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td>( t )</td>
</tr>
<tr>
<td>Position</td>
<td>( x )</td>
</tr>
<tr>
<td>Velocity</td>
<td>( \dot{x} )</td>
</tr>
<tr>
<td>Acceleration</td>
<td>( \ddot{x} )</td>
</tr>
<tr>
<td>Force</td>
<td>( F )</td>
</tr>
</tbody>
</table>

Parameter | Dimensionless Value |
-----------|---------------------|
Mass       | \( m \)             |
Viscous friction | \( b \)     |
Coulomb friction | \( c \)       |

\[
\mu \ddot{x}(\tau) + \beta \dot{x}(\tau) + \sigma \text{sgn}\dot{x}(\tau) = \varphi_H(\tau) + \varphi_A(\tau)
\]

\[
\varphi_A(\tau) = -[\xi(h)] - \frac{1}{2} \quad \forall \tau \in [h; h + 1[, \ h \in \mathbb{N}
\]
Effects of Parameters Variations

\[ \beta := \frac{b}{KT} \]

\[ \sigma := \frac{c}{K\Delta} \]
Effects of Parameters Variations

\[ \beta := \frac{b}{KT} \]

\[ \sigma := \frac{c}{K \Delta} \]
Effects of Parameters Variations

\[ \beta := \frac{b}{KT} \]

\[ \sigma := \frac{c}{K\Delta} \]
Energetic Analysis of Digital Springs

Digital (discretized and quantized) spring
Energy Balance

- By considering power $P_d$ dissipated because of friction and power $P_g$ generated by energy "leaks"

$$\int_{\tau_0}^{\tau_1} \varphi_H(\tau) \dot{\xi}(\tau) d\tau = H_T(\tau_1) - H_T(\tau_0) + \int_{\tau_0}^{\tau_1} P_d(\tau) d\tau - \int_{\tau_0}^{\tau_1} P_g(\tau) d\tau$$

- Therefore the haptic interaction is passive if the storage function exists and:

$$\int_{\tau_0}^{\tau_1} P_d(\tau) d\tau \geq \int_{\tau_0}^{\tau_1} P_g(\tau) d\tau \quad \forall \tau_1 \geq \tau_0$$
Storage Function (Energy) for a Quantized Spring

- We refer to the quantized, time-continuous spring:

\[
\varphi_Q(\tau) = -\lfloor \xi(\tau) \rfloor - \frac{1}{2} \quad \xi = \lfloor \xi \rfloor + \rho \quad 0 \leq \rho < 1
\]

- Storage function:

\[
\mathcal{H}_T(\xi, \dot{\xi}) = \frac{1}{2} \mu \dot{\xi}^2 + \frac{1}{2} \left( \xi^2 + \rho(\xi) - \rho^2(\xi) \right)
\]
Energy Leaks

The generated energy is computed as the deviation of the digital spring from its quantized, time-continuous counterpart:

\[ E_g(\tau_0, \tau_1) = \int_{\tau_0}^{\tau_1} P_g(\tau) d\tau = \int_{\tau_0}^{\tau_1} [\varphi_A(\tau) - \varphi_Q(\tau)] \dot{\xi}(\tau) d\tau \]

\[ E_g(h, \tau_1) = \int_{h}^{\tau_1} (\lfloor \xi(\tau) \rfloor - \lfloor \xi(h) \rfloor) \dot{\xi}(\tau) d\tau \quad h \leq \tau_1 < h + 1 \]

\[ E_{gz} := \int_{h}^{\tau_1} (\xi(\tau) - \xi(h)) \dot{\xi}(\tau) d\tau \quad \text{Effect of time-discretization} \]

\[ E_{gq} := \int_{h}^{\tau_1} (\rho(\tau) - \rho(h)) \dot{\xi}(\tau) d\tau \quad \text{Effect of quantization} \]
Energy Leaks

- Term due to discretization:

\[ E_{gz} = \frac{1}{2} \left( \xi(\tau_1) - \xi(h) \right)^2 \]

- The term due to quantization can be rewritten as:

\[ E_{gq} = \left( \rho(h) - \frac{1}{2} \right) \left( \left\lfloor \xi(\tau_1) \right\rfloor - \left\lfloor \xi(h) \right\rfloor \right) - \frac{1}{2} \left( \rho(\tau_1) - \rho(h) \right)^2 \]

- Compute the maximum with respect to \( \rho \):

\[ E_{gq} \leq \max_{\rho(h), \rho(\tau_1)} \frac{1}{2} \left| \left\lfloor \xi(\tau_1) \right\rfloor - \left\lfloor \xi(h) \right\rfloor \right| = \frac{1}{2} \left| \xi(\tau_1) - \xi(h) \right| \]

- Finally:

\[ E_g(h, \tau_1) \leq \frac{1}{2} \left( \xi(\tau_1) - \xi(h) \right)^2 + \frac{1}{2} \left| \xi(\tau_1) - \xi(h) \right| \]
Energy Dissipation

- Total dissipation:

\[ E_d(h, \tau_1) = \int_{h}^{\tau_1} P_d(\tau) d\tau = \int_{h}^{\tau_1} \beta \ddot{\xi}^2(\tau) d\tau + \int_{h}^{\tau_1} \sigma |\dot{\xi}(\tau)| d\tau \]

- From the triangular inequality:

\[ \int_{h}^{\tau_1} \sigma |\dot{\xi}(\tau)| d\tau \geq \sigma |\xi(\tau_1) - \xi(h)| \]

- Cauchy-Schwarz inequality:

\[ \int_{h}^{\tau_1} \beta \ddot{\xi}^2(\tau) d\tau \geq \beta \left( \frac{\xi(\tau_1) - \xi(h)}{\tau_1 - h} \right)^2 \]

- Finally:

\[ E_d \geq \beta \left( \frac{\xi(\tau_1) - \xi(h)}{\tau_1 - h} \right)^2 + \sigma |\xi(\tau_1) - \xi(h)| \]
Passivity Condition

\[ E_g(h, \tau_1) \leq E_d(h, \tau_1) \quad \forall \tau_1 \in [h, h + 1[, \forall h \in \mathbb{N} \]

- The velocity is a continuous function, the mean value theorem holds:

\[ |\dot{\xi}(\tau_1) - \dot{\xi}(h)| = |\dot{\xi}(\tau)|(\tau_1 - h) \leq |\dot{\xi}(\tau)| \quad h < \tau < \tau_1 \]

- And by means of previous bounds:

\[
\begin{align*}
E_g(h, \tau_1) &\leq \frac{1}{2}(\xi(\tau_1) - \xi(h))^2 + \frac{1}{2}|\xi(\tau_1) - \xi(h)| \\
E_d(h, \tau_1) &\geq \beta \frac{(\xi(\tau_1) - \xi(h))^2}{\tau_1 - h} + \sigma |\xi(\tau_1) - \xi(h)| \\
\end{align*}
\]

\[ \left( \beta - \frac{1}{2} \right) + \frac{\sigma - 1}{2} \geq 0 \quad \forall \tau > 0 \]
Passivity Condition

The “passivity” of the overall system is preserved if the intrinsic dissipation of the device is always greater than the energy generated by the non-idealities:

\[
\left( \beta - \frac{1}{2} \right) + \frac{\sigma - \frac{1}{2}}{\dot{\xi}(\tau)} \geq 0 \quad \forall \tau > 0
\]

Without Coulomb dynamic friction and position quantization, the Colgate inequality is obtained:

\[
\beta > \frac{1}{2} \quad \quad b > \frac{KT}{2}
\]
Interpretation

$$\left( \beta - \frac{1}{2} \right) + \frac{\sigma - \frac{1}{2}}{\dot{\xi}(\tau)} \geq 0$$

$$\sigma \geq \frac{1}{2} \implies c \geq \frac{K\Delta}{2}$$
Simulations

\[ \dot{\xi}_0 = 20 \]
Experiments on a 1DOF setup

(0.090405,13.1881)

(0.90405,26.3762)
Ok, I trust you... But what about real devices?

- Most devices operate in the locally stable/unstable region (viscous friction is usually very very very very small)

<table>
<thead>
<tr>
<th>Device</th>
<th>$m$ [Kg]</th>
<th>$b$ [Ns/m]</th>
<th>$c$ [N]</th>
<th>$\Delta$ [um]</th>
<th>$T$ [ms]</th>
<th>$K$ [N/m]</th>
<th>$\mu$</th>
<th>$\beta$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Omega</td>
<td>0.220</td>
<td>0.01</td>
<td>0.147</td>
<td>10</td>
<td>0.33</td>
<td>14500</td>
<td>136.6</td>
<td>0.002</td>
<td>1.01</td>
</tr>
<tr>
<td>Delta</td>
<td>0.250</td>
<td>0.01</td>
<td>0.883</td>
<td>30</td>
<td>0.33</td>
<td>14500</td>
<td>155.2</td>
<td>0.002</td>
<td>2.03</td>
</tr>
<tr>
<td>Impulse Engine</td>
<td>0.032</td>
<td>0.02</td>
<td>0.024</td>
<td>31.4</td>
<td>0.2</td>
<td>800</td>
<td>1007.8</td>
<td>0.13</td>
<td>0.97</td>
</tr>
<tr>
<td>Phantom 1.0</td>
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<td>0.005</td>
<td>0.038</td>
<td>29.1</td>
<td>1</td>
<td>1015</td>
<td>70.55</td>
<td>0.004</td>
<td>1.29</td>
</tr>
<tr>
<td>Toolhandle</td>
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<td>0.001</td>
<td>0.034</td>
<td>20.1</td>
<td>1</td>
<td>3125</td>
<td>38.19</td>
<td>0.0003</td>
<td>0.54</td>
</tr>
<tr>
<td>Human Operator</td>
<td>0.150</td>
<td>4.8</td>
<td></td>
<td></td>
<td></td>
<td>600</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- ... And the human operator helps to stabilize the virtual walls!!!

- Work in progress: how to use this analysis to stabilize the rendering of higher stiffnesses?
CASY COLLOQUIUM - END
Describing Function Analysis

\[ \Phi_A(M, \omega) = G(M, \omega) \Xi(M, \omega) = \left[ -\mu \omega^2 + j \left( \beta \omega + 4 \frac{\sigma}{\pi M} \right) \right] \Xi(M, \omega) \]
Describing Function Analysis

\[ D(M) = \frac{2}{\pi M} + \frac{4}{\pi M^2} \sum_{l=1}^{[M]} \sqrt{M^2 - l^2} \]

\[ D_1(M) \]

\[ D(M) \]
Describing Function

- Two families of solutions $(M<1$ and $M>1)$
Workpackage WP-8: **Telemanipulation systems**

**Activities:**
- Development of robotic systems for telemanipulation tasks
  - Implementation of different control algorithms
    - Tools for rapid prototyping
  - Estimation of contact impedance and transparency improvement
- Development of a geometrical method for representing scattering phenomena in distributed PH systems
Laboratory Setup - 2

- 1 DOF Master – Slave system:
  - Master: rotational or linear actuator
  - Slave: linear actuator